Name: _____

Student #:_____

Mark: _____/150

University of Idaho

CSCI 395– Mid-term 1

This is a closed book test.

No electronic device (including calculators or smart watches) permitted.

Please ensure you have all pages before you begin.

Write your answers on the question paper in the spaces provided. Use the reverse side and the last page for rough work. The total time allowed is 80 minutes.

Question 1: Define what f(x) $\in$ O(g(x)) means

You know there will be a question like this!!! (1-7)

/5

Question 2: Prove $8x^2 + 5x + 9 \in O(x^2)$ from the definition without using limits.

/10

Question 3: Define what $f(x) \in \Omega(g(x))$ means

/5

Question 4: Prove $8x^2 + 5x + 9 \in \Omega(x^2)$ from the definition without using limits.

/10

Question 5: Define what $f(x) \in \Theta(g(x))$ means

/5

Question 6: Prove $8x^2 + 5x + 9 \in \Theta(x^2)$ from the definition without using limits. (You can assume the parts you have previously proven are true).

/5

Question 7: Prove $8x^2 + 5x + 9 \in \Theta(x^2)$ using limits.

/5

Question 8: For each of the following **circle either True or False**. Note: If it is not ALWAYS true then it is false.

There might be questions like this

True **False** If $f(x) \in O(g(x))$ THEN $g(x) \in \Omega(f(x))$

True **False** If $f(x) \in O(g(x))$ AND $g(x) \in O(h(x))$ THEN $f(x) \in O(h(x))$

/5

True **False** If $f(x) \in O(g(x))$ THEN $g(x) \in O(f(x))$

True **False** If $f(x) \in \Theta(g(x))$ THEN $g(x) \in \Theta(f(x))$

True **False** This equation is linear: $Q(n) = Q(n-1) * Q(n-2)$

Question 9: Write **O, Ω or Θ** on each of the following blank lines to indicate who each of the functions compares to the other.

A. x^2 $\in$_____(x^3)

B. $x^{0.001}$ $\in$_____(log x)

/10

C. 2^n $\in$_____ (n!)

D. log x^2 $\in$_____(($\log x)^2$)

E. n^2 $\in$_____(n log n)

Question 10: Consider this algorithm for matrix multiplication:

ALGORITHM *MatrixMultiplication*$(A[0..n-1, 0..n-1], B[0..n-1, 0..n-1])$

```
//Multiplies two n-by-n matrices by the definition-based algorithm
//Input: Two n-by-n matrices A and B
//Output: Matrix C = AB
for i ← 0 to n − 1 do
    for j ← 0 to n − 1 do
        C[i, j] ← 0.0
        for k ← 0 to n − 1 do
            C[i, j] ← C[i, j] + A[i, k] * B[k, j]
return C
```

The formula expressing operation count as function of size n is

There should be at least one sigma question from an algorithm. If it is contains formula you have not memorized, then I will give it to you. **/10**

Solve the formula precisely. Note: this formula might help:

$\Sigma_{1 \le i \le n} i^2 = 1^2+2^2+ \cdots +n^2 = n(n+1)(2n+1)/6 \approx n^3/3 \in \Theta(n^3)$

11. For a Fibonacci sequence,

F(n) = F(n-1) + F(n-2).

F(0) = 0.

F(1) = 1.

Use the characteristic equation to derive the formula for F(n).

You know this question will be there.

I will probably also ask you an easy one using the characteristic equation.

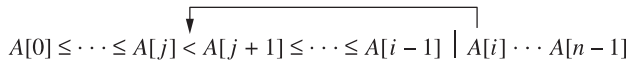
Be careful. I might ask you a trick question.

/20

12. **Sort the letters {S, O, R, T, I, N, G} into alphabetical order using *Insertion Sort*. Show your steps in the table. (You might not need all the rows in the table.)**

ALGORITHM *InsertionSort*(A[0..n − 1])
//Sorts a given array by insertion sort
//Input: An array A[0..n − 1] of n orderable elements
//Output: Array A[0..n − 1] sorted in nondecreasing order
for i ← 1 **to** n − 1 **do**
    v ← A[i]
    j←i−1
    while j ≥ 0 **and** A[j] > *v* **do**
        A[j + 1] ← A[j]
        j←j−1
    A[j + 1] ← *v*

Recall:



Initially →	S	O	R	T	I	N	G
1)							
2)							
3)							
4)							
5)							
6)							
7)							
8)							

/10

At least one of the sorting algorithms you will have to do manually

13. Consider these items you want to use to fill a knapsack that holds 12 pounds:

Item	Value	Weight
A	5	3
B	8	4
C	7	6

How many combinations will you need to consider for an exhaustive search? **/2**

Fill in the table below with all possible combinations, and their weight and value. An X indicates that the item is included in this calculation. The first one is done for you. You might not need all rows:

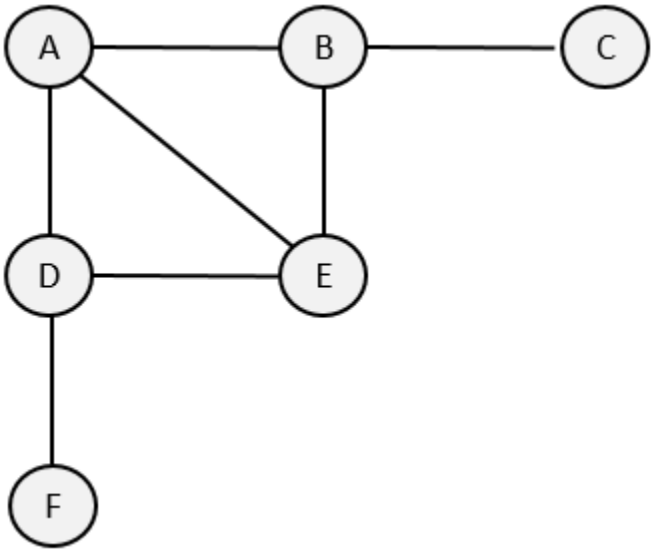
A	B	C	Value	Weight
		X	7	6

/16

What is the best combination of items to pack? (What is its value?) **/2**

There will be an exhaustive search!

14. Do a Depth First Search of this graph. Indicate in what order each vertex is placed on the stack and removed from the stack. Remember, all ties are broken alphabetically.



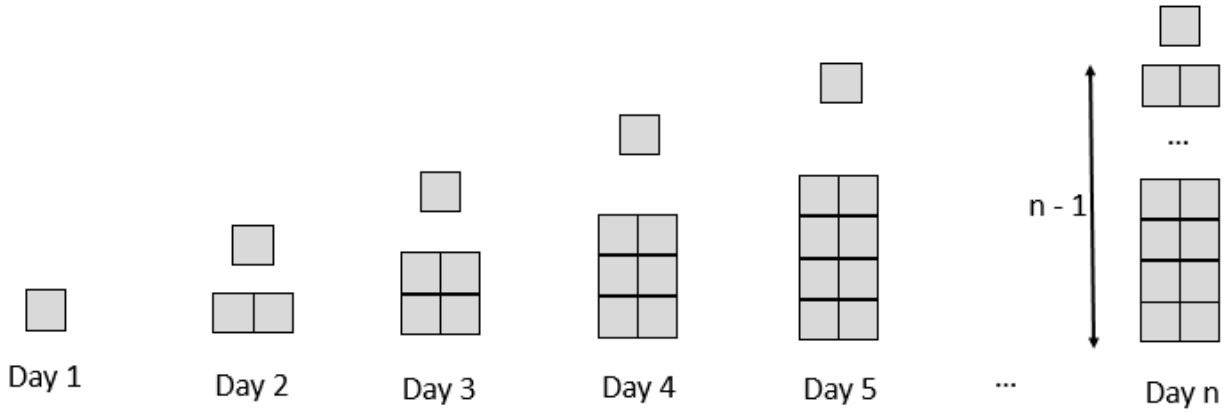
/10

There will probably be a DFS and/or BFS!

/5

15. Do a Breadth First Search of the same graph. Indicate the order in which each vertex is placed in the queue.

16. You have been hired by the University of Idaho to make lower case 'l's using prefabricated blocks. Each l is 2 blocks wide with a single block at the top for the dot. The height of each l gets one taller each day. The diagram below shows the result of the first n days.



The cost of all the blocks you will need to complete day n can be given by the formula:

Day(n) = Day(n-1) + 2(n-1) + 1

Note: Day n included the blocks used that day as well as all the blocks used on previous days.

Use forward substitution to determine how may blocks you will have used at the end of each of the first seven days.

Day	Blocks used so far
0	0
1	
2	
3	
4	
5	
6	
7	

/5

There will be a creative question, but it will be based off something we was in class

Use backwards substitution to derive a formula for the number of blocks you will have to order to stay in production until the end of Day(n).

$$Day(n) = Day(n-1) + 2(n-1) + 1$$

/10

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