

CS 395 – Analysis of Algorithms

Chapter 6 – Transform-and-Conquer

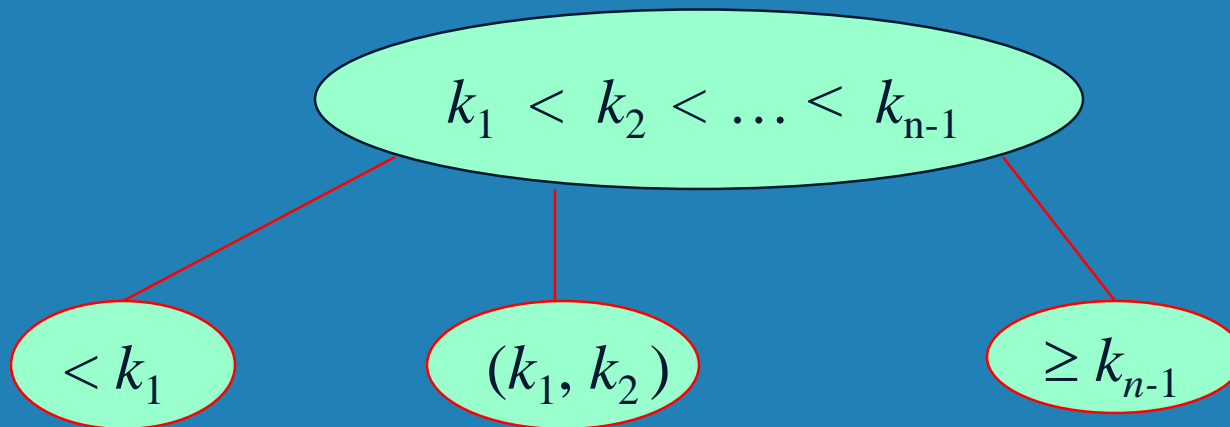
Read 6.3

- **Balanced Search Trees**
 - AVL Trees
 - **2-3 Trees**

Multiway Search Trees

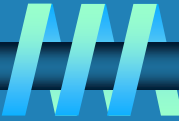
Definition A *multiway search tree* is a search tree that allows **more than one key** in the same node of the tree.

Definition A node of a search tree is called an *n-node* if it contains **$n-1$ ordered keys** (which divide the entire key range into **n intervals** pointed to by the node's n links to its children):



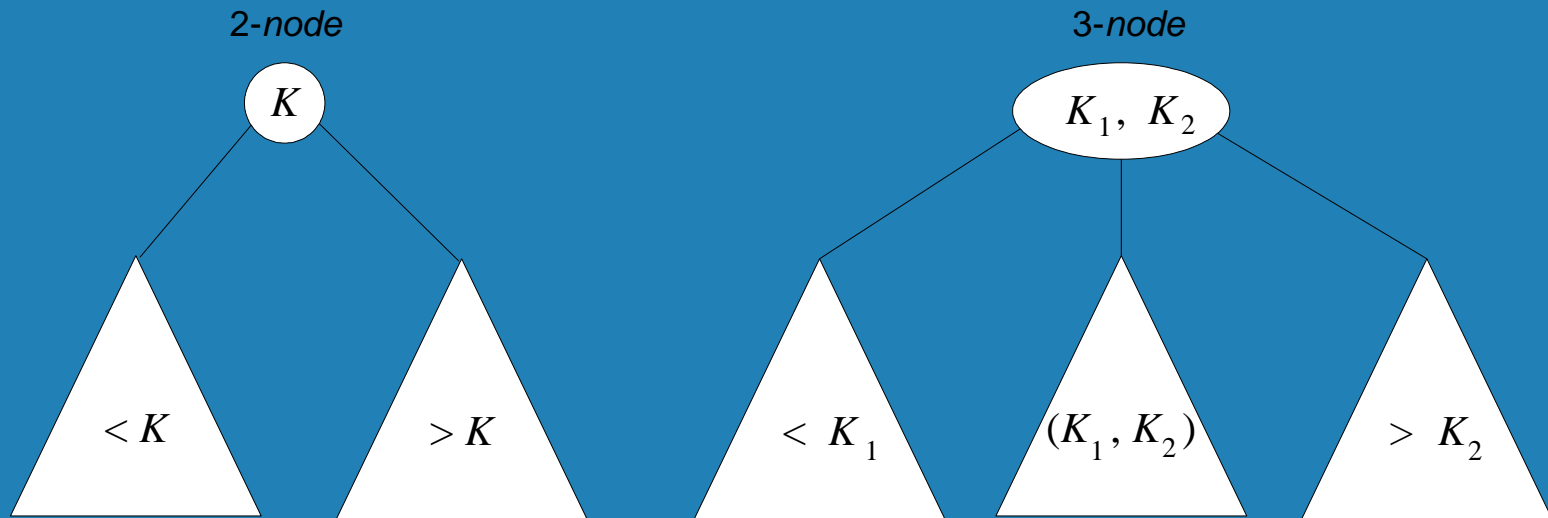
Note: Every node in a classical binary search tree is a 2-node

2-3 Tree



Definition A *2-3 tree* is a search tree that

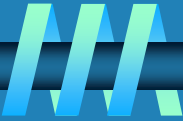
- ∞ may have **2-nodes** and **3-nodes**
- ∞ **height-balanced** (all leaves are on the same level)



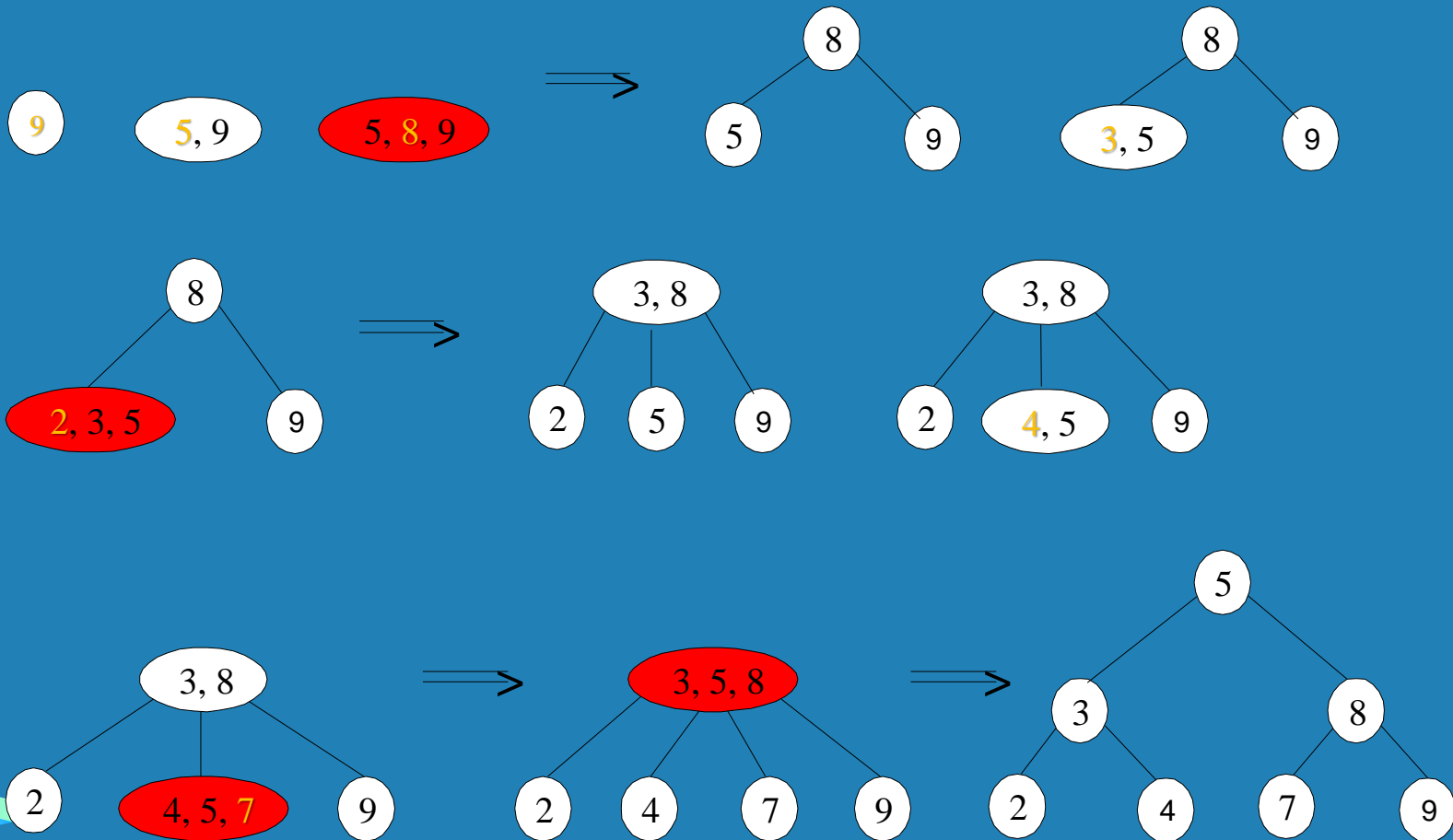
A 2-3 tree is constructed by successive insertions of keys given, with a new key always inserted into a leaf of the tree.

If the leaf is a 3-node, it's split into two with the middle key promoted to the parent.

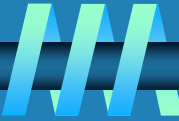
2-3 tree construction – an example



Construct a 2-3 tree the list 9, 5, 8, 3, 2, 4, 7



Analysis of 2-3 trees



- $\log_3(n + 1) - 1 \leq h \leq \log_2(n + 1) - 1$
- Search, insertion, and deletion are in $\Theta(\log n)$
- The idea of 2-3 tree can be generalized by allowing more keys per node
 - 2-3-4 trees (**we are skipping those**)
 - B-trees (**also skipped**)

