Useful Summation Formulas and Rules

$$\sum_{l \leq i \leq u} 1 = 1 + 1 + \ldots + 1 = u - l + 1$$

In particular: $\displaystyle\sum_{1 \leq i \leq n} 1 = n - 1 + 1 = n \in \Theta(n)$

$$\sum_{1 \leq i \leq n} i = 1 + 2 + \ldots + n = \frac{n(n+1)}{2} \approx \frac{n^2}{2} \in \Theta(n^2)$$

$$\sum_{1 \leq i \leq n} i^2 = 1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{n^3}{3} \in \Theta(n^3)$$

$$\sum_{0 \leq i \leq n} a^i = 1 + a + \ldots + a^n = \frac{a^{n+1} - 1}{a - 1}, \quad a \neq 1$$

In particular: $\displaystyle\sum_{0 \leq i \leq n} 2^i = 2^0 + 2^1 + \ldots + 2^n = 2^{n+1} - 1 \in \Theta(2^n)$

$$\sum (a_i + b_i) = \sum a_i + \sum b_i$$

$$\sum (c a_i) = c \sum a_i$$

$$\sum_{l \leq i \leq u} a_i = \sum_{l \leq i \leq m} a_i + \sum_{m+1 \leq i \leq u} a_i$$