Midterm Exam formulas

Formulas for basic calculations

$$\bar{x} = \frac{\sum x_i}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n-1}$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{s^2}$$

Empirical rule: $68\%: \bar{x} \pm 1s,\ 95\%: \bar{x} \pm 2s,\ 99.7\%: \bar{x} \pm 3s$

Probability rules

(1) Validity: $0 \leq P(A) \leq 1$
(2) Total: $\sum P(A_i) = 1$
(3) Complement: $P(A') = 1 - P(A)$
(4) Addition: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(5) Multiplication: $P(A \cap B) = P(A)P(B)$ only if A and B are independent

Generic probabilities

$$E(X) = \mu = \sum x \cdot p(x) = x_1 p(x_1) + \cdots + x_n p(x_n)$$

$$V(X) = \sigma^2 = \sum (x - EX)^2 \cdot p(x) = (x_1 - EX)^2 p(x_1) + \cdots + (x_n - EX)^2 p(x_n)$$

$$SD(X) = \sigma = \sqrt{V(X)}$$

Binomial distribution: $X \sim bin(n, p)$

$$P(X = x) = \binom{n}{x} p^x q^{n-x} \text{ with } q = 1 - p, \binom{n}{x} = \frac{n!}{x!\,(n-x)!}, EX = np \quad VX = npq \quad SDX = \sqrt{npq}$$

Poisson distribution: $X \sim pois(\mu)$

$$P(X = x) = \frac{\mu^x e^{-\mu}}{x!} \quad EX = \mu \quad VX = \mu \quad SDX = \sqrt{\mu}$$

Normal distribution: $X \sim N(\mu, \sigma)$

$$z = \frac{x - \mu}{\sigma}$$

Central Limit Theorem for distributions of sample statistics

Distribution of sample mean

$$\overline{X} \sim N(\mu, se) \text{ with } se = \frac{\sigma}{\sqrt{n}},\ z = \frac{\overline{X} - \mu}{se}$$

Distribution of sample proportion

$$\hat{p} \sim N(p, se) \text{ with } se = \sqrt{\frac{pq}{n}},\ q = 1 - p,\ z = \frac{\hat{p} - p}{se}$$

Distribution of sample total

$$\hat{\tau} \sim N(\tau, se) \text{ with } se = \sigma\sqrt{n},\ \tau = n\mu,\ z = \frac{\hat{\tau} - \tau}{se}$$